Dissipation and Tunnelling in Quantum Hall Bilayers

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We discuss the interplay between transport and intrinsic dissipation in quantum Hall bilayers, within the framework of a simple thought experiment. We compute, for the first time, quantum corrections to the semiclassical dynamics of this system. This allows us to re-interpret tunnelling measurements on these systems. We find a strong peak in the zero-temperature tunnelling current that arises from the decay of Josephson-like oscillations into incoherent charge fluctuations. In the presence of an in-plane field, resonances in the tunnelling current develop an asymmetric lineshape.

Transport in quantum Hall bilayers has been the subject of much recent interest[1, 2, 3, 4, 5, 6, 7]. The bilayers consist of two closely-spaced parallel two-dimensional electron layers in a double quantum well. If the Landau level fillings of the layers are $\nu_1 = \nu_2 = 1/2$, then Coulomb interactions between the layers drive a transition to a ground state in which the bilayer as a whole exhibits the quantum Hall effect[1]. This ground state is believed to have a broken U(1) symmetry[8]; there is a macroscopically coherent phase associated with the electrons' layer degree of freedom. The ground state can be viewed as an easy-plane ferromagnet[3] or, equivalently, as an excitonic condensate[4, 9]. There are also analogies with Josephson junctions[8].

A series of remarkable experiments[2, 7, 10] have been used to probe the internal degrees of freedom of this strongly correlated system. They show evidence for interlayer coherence and the linearly dispersing Goldstone mode resulting from the broken U(1) symmetry[2], and also, most recently, "excitonic" superfluidity[7].

There still remain questions concerning the interlayer tunnelling spectrum of the bilayers. One prominent feature in the IV characteristic is a sharp peak in the tunnelling current for small biases (between 10 and 100 μV)[2]. At low temperatures, reducing the bias leads to a sharp *rise* in the tunnelling current. This increase is cut off below 10 μV so that the current falls to zero at zero bias. Existing theories for interlayer tunnelling are restricted to the classical limit of the underlying spin model, and do not produce this feature at the low temperatures relevant to experiment ($k_B T \sim 2\mu$eV).

In this paper we study interlayer tunnelling within the framework of a simple "thought experiment". We follow the relaxation of an initial charge imbalance across the bilayer. Including quantum $(1/S)$ corrections to the dynamics of the pseudospin model of the bilayer[3], we see that electron tunnelling across the bilayer generates density waves. This quantum dissipative process leads to a zero-temperature tunnelling current of the form $I \propto 1/V$ for a bias voltage V above a threshold V_0; this is consistent with experimental measurements. The long-

wavelength density fluctuations in the bilayer have an energy gap Δ_{sw}; in the absence of disorder the threshold V_0 corresponds to an energy of this order. Introducing an in-plane magnetic field to our calculations causes the small bias feature to develop into an asymmetric resonant peak in the tunnelling current, similar to that reported in [2]. Below the threshold V_0, we find that the intrinsic dissipation causes any macroscopically coherent charge oscillations to decay in time, consistent with the absence of Josephson-like oscillations in experiments.

We work in the pseudospin picture of the bilayer[3]. The charge imbalance on the bilayer is given by the z-component of the magnetisation of the pseudospins. The system is a ferromagnet due to Coulomb exchange. Since a z component in the magnetisation incurs a capacitative energy cost, the ferromagnet has easy-plane anisotropy.

We use a model of discrete spins on a lattice, beginning with a spin-1/2 system in which each spin represents a local single-particle state within the lowest Landau level. We work with a large-S version of this system. This large-S generalisation may be treated as a coarse-graining procedure. The limit of large S is the classical limit for the spin system. We work with the Hamiltonian:

$$\frac{H}{2S} = -\frac{\rho_E}{2} \sum_{\langle ij \rangle} \vec{m}_i \cdot \vec{m}_j + \frac{D}{4} \sum_i (m_i^z)^2 - \frac{\Delta_{\mathrm{SAS}}}{2} \sum_i m_i^x \quad (1)$$

where $\vec{S}_i = S\vec{m}_i = S(m_i^x, m_i^y, m_i^z)$ is the spin operator on site i of a square lattice with spacing $c_0 = \sqrt{2\pi} l_B$ where $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length. The exchange ρ_E and the strength of the on-site repulsion D were derived from microscopic considerations by Moon *et al.*[3]. The tunnelling between the layers enters the problem through Δ_{SAS}: the splitting of the "bonding" and "anti-bonding" single-particle states in the double well. In our thought experiment, a gate is used to control the charge imbalance on the bilayer: this adds a term $H_V = -SV \sum_i m_i^z$ to the Hamiltonian. Typical values for the model parameters in physical bilayer systems are $l_B \simeq 20$nm, $\Delta_{\mathrm{SAS}} \simeq 90\mu$K, $\rho_E \simeq 0.5$K, $D \simeq 30$K.

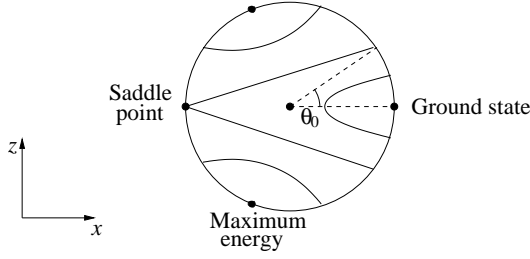We consider the properties of the ferromagnetic phase

FIG. 1: Mean field (classical) spin trajectories are equipotentials on the surface of the spin sphere.

of this model, which is believed to describe the experimental systems. In particular, the observation of a linearly dispersing peak in the tunnelling conductance[2] indicates that the experimentally accessible state has ferromagnetic order — the peak arises from the Goldstone mode of the system. We therefore ignore the quantum disordered phase that exists for $D/\rho_E S^2 \gg 1$ [11].

We will now outline the dynamics of the classical ferromagnetic system before we discuss the quantum effects which are responsible for the existence of a dissipative tunnelling current. In the classical system, a spatially uniform spin configuration will remain uniform forever. The total magnetisation precesses along a trajectory of constant energy. We sketch the trajectories in Fig. 1 for $\Delta_{SAS} \ll D$ which is the physically relevant regime.

Near the (x-polarised) ground state, the precession frequency is given by $\Delta_{sw}/\hbar$ where $\Delta_{sw} = [\Delta_{SAS}(\Delta_{SAS} + D)]^{1/2}$ is the energy gap for spinwave excitations (density waves of charge imbalance across the bilayer). As the magnetisation precesses along these trajectories, the charge imbalance on the bilayer oscillates around zero. Far away from the ground state, the spin precesses around one of the two maximal energy states which lie close to the S^z axis. This yields a Josephson-like alternating current $I \simeq e\Delta_{SAS}\cos(eVt/\hbar)$ where V is the voltage across the bilayer due to capacitive charging. We stress that this is valid only for large charge imbalance (large V).

Trajectories through the saddle point at $\vec{m} = (-1,0,0)$ mark the boundary between oscillations around the ground state and oscillations around the maximal energy states. The saddle point trajectory crosses the xz-plane at three points, as shown in Fig. 1. The angle θ_0 satisfies $\cos\theta_0 = 1 - 2(\Delta_{SAS}/D)$. It corresponds to a voltage difference of $V_0 = 2\Delta_{sw}/e$ across the layers.

In our thought experiment, we imagine using a gate to induce a uniform charge imbalance on the bilayer. In the spin picture, the magnetisation is tilted out of the easy (xy) plane. The bias is then instantaneously removed and the bilayer finds itself in a highly excited state.

The resulting behaviour depends crucially on whether the initial gate voltage V is above or below the saddle point value V_0. We treat the two regimes in separate

perturbation theories. In both calculations, we expand around the classical limit using spinwave theory in a $1/S$ expansion. The leading terms in this expansion result in the leading terms in the I-V relation.

We begin by considering the situation for initial energies above the saddle point ($V > V_0$). When the gate voltage is removed, the charge imbalance results in a potential difference across the bilayer equal to the original gate voltage V. As discussed above, this causes an alternating current with a frequency ($eV/\hbar$). In this regime, we can treat the tunnelling Δ_{SAS} perturbatively.

The quantum spin system is distinct from the classical one in that spatially uniform oscillations do not persist — the tunnelling term breaks the global spin rotation symmetry so that long-wavelength modes are no longer protected from decay by Goldstone's theorem. The uniform mode decays by transferring energy into spin waves with finite wavevectors. The magnetisation falls to a lower trajectory on the spin sphere, corresponding to a net transfer of charge across the bilayer. We can therefore obtain the d.c. tunnelling current by calculating the rate of this dissipative process.

It is straightforward to derive a bosonic spinwave theory for our model. We use the Holstein-Primakov representation: $S_j^x + iS_j^y = (2S - a_j^\dagger a_j)^{1/2} a_j$, $S_j^z = S - a_j^\dagger a_j$. Expanding around $m^z = (eV/D) = \sin\theta$, the quadratic part of the Hamiltonian is then easily diagonalised in the Fourier basis to give

$$H_{\theta>\theta_0}^{(0)} = (D\sin\theta)\delta n_{q=0} + \sum_{\boldsymbol{q}\neq 0} \varepsilon_{\boldsymbol{q}} \alpha_{\boldsymbol{q}}^\dagger \alpha_{\boldsymbol{q}} \qquad (2)$$

where the $\alpha_{\boldsymbol{q}}$ are bosonic operators describing the spinwave modes. The spinwave dispersion is given by $\varepsilon_{\boldsymbol{q}} = [\rho_E\gamma(\boldsymbol{q})(D + \rho_E\gamma(\boldsymbol{q}))]^{1/2}$ where $\gamma(\boldsymbol{q}) = 4 - 2\cos(q_x c_0) - 2\cos(q_y c_0)$. The dispersion is linear and gapless at small q. (Tunnelling should produce a small energy gap but this does not affect the perturbative calculation we describe here.) The spinwave velocity is $v = l_B(2\pi D\rho_E)^{1/2}$. Observe that the $q = 0$ mode has been singled out in the Hamiltonian, and its energy is not given by the long wavelength limit of $\varepsilon_{\boldsymbol{q}}$. The quanta of this mode carry $S^z = 1$ while spin waves with finite wavevector have $S^z = 0$.

Starting from an initial state $|i\rangle$, the energy dissipation rate is given by

$$\Gamma = \partial_t \left\langle i \left| e^{iHt} \sum_{\boldsymbol{q}\neq 0} \varepsilon_{\boldsymbol{q}} \alpha_{\boldsymbol{q}}^\dagger \alpha_{\boldsymbol{q}} e^{-iHt} \right| i \right\rangle \qquad (3)$$

The dissipation arises from the destruction of one quantum in the $q = 0$ mode, and the generation of multiple spinwaves during tunnelling across the bilayer. To leading order in $1/S$, a pair of spin waves with opposite momenta is excited (Fig. 2a). The relevant vertex is:

$$H_{\theta>\theta_0}^{(1)} = (-\tilde{\Delta}_{SAS}/8)e^{-i\phi_{q=0}} \sum_{\boldsymbol{q}} \gamma_{2,\boldsymbol{q}} \alpha_{\boldsymbol{q}}^\dagger \alpha_{-\boldsymbol{q}}^\dagger + \text{h.c.} \quad (4)$$
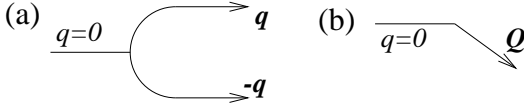
FIG. 2: Decay processes for $V > V_0$. (a) dominant process for Q=0: $\varepsilon_{\boldsymbol{q}} + \varepsilon_{-\boldsymbol{q}} = eV$. (b) Decay process at finite Q: $\varepsilon_{\boldsymbol{Q}} = eV$.
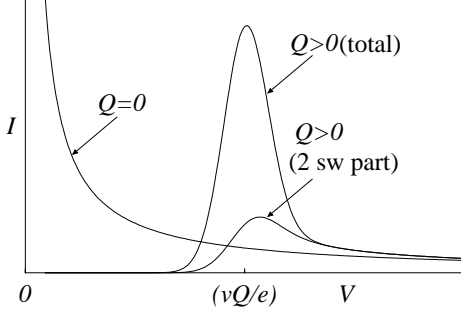

FIG. 3: Plots of the current I (arbitrary units) at $Q = 0$ and at finite $Q = 0.5(4\pi\rho_E S/v)$ so that $vQ \sim 150\mu V$. The two spinwave contribution (2sw) to the current gives rise to the asymmetric peak, as shown. The contributions at finite Q have been broadened by a Gaussian distribution for Q of width $\sigma_Q = 0.1(4\pi\rho_E S/v)$.

in which $\tilde{\Delta}_{\text{SAS}} = \Delta_{\text{SAS}} \exp(-\sqrt{D/\rho_E S^2})$ is a renormalised tunnelling amplitude, and $\gamma_{2,\boldsymbol{q}} = \cos\theta[(u_{\boldsymbol{q}} + v_{\boldsymbol{q}})^2 + 2\sin\theta \sec^2\theta - (u_{\boldsymbol{q}} - v_{\boldsymbol{q}})^2 \sec^4\theta]$ where $u_{\boldsymbol{q}}$ and $v_{\boldsymbol{q}}$ are coherence factors: $(u_{\boldsymbol{q}} + v_{\boldsymbol{q}})^2 = (D + \rho_E \gamma(\boldsymbol{q}))/\varepsilon_{\boldsymbol{q}}$ with $u_{\boldsymbol{q}}^2 - v_{\boldsymbol{q}}^2 = 1$. The θ-dependence arises because the x-component of the pseudospin depends on its angle with the easy plane, θ, as well as on the azimuthal angle ϕ. This dependence is weak for $\theta \ll 1$ when the charge imbalance is small compared to the Landau level filling.

We now calculate the power dissipation Γ for a given initial voltage V; the steady-state tunnelling current density at a bias V is $I = \Gamma/VL^2 S$. The result is:

$$\frac{\Gamma_{\theta > \theta_0}}{L^2 S} = I_{\theta > \theta_0} V = \frac{D\tilde{\Delta}_{\text{SAS}}^2}{32\pi\rho_E l_B^2 \hbar S} [1 + X(\theta)]^2 \quad (5)$$

where $X(\theta) = (\sec\theta - 1)(\sec^2\theta - 2\sec\theta - 1)$ is small for a small charge imbalance ($\theta \ll 1$). Thus, the power dissipation into spinwave pairs is *independent* of the voltage, and so the tunnelling current has a $1/V$ divergence. This should be cut off at low bias when the bias V becomes comparable to V_0 and perturbation theory breaks down. This contribution to the tunnelling current is significant at low temperatures: it is our main result, consistent with the experimental observation[2] of a peak at a bias close to the spinwave gap. Its contribution to the tunnelling current density is shown in Fig. 3.

We note that the region of the experimental sample in which the tunnelling current flows remains an open question[4]. Estimating the power dissipated in the experiments[2] to be $10^{-16}W$, and neglecting renormal-

isation of the tunnelling (so $\tilde{\Delta}_{\text{SAS}} = \Delta_{\text{SAS}}$), we obtain an estimate of $50\mu m^2$ for this area. This estimate is increased if we use a renormalised Δ_{SAS}. The result is consistent with tunnelling taking place near the contacts to the bilayer, rather than over its entire area. In the experimental data of [2], the current decays more slowly than $1/V$ away from the resonance, so that the power increases with increasing applied voltage. We attribute this increase to other dissipative channels, due to disorder or finite temperature. Processes at higher order in $1/S$ also affect the current in this way (see (7) below), but are not large enough to explain the differences between theory and experiment.

It should be noted that the dissipation rate (5) is averaged over the period of the Josephson oscillations. As with other theories in which the tunnelling is treated perturbatively[4, 6], we also expect an oscillatory a.c. component with frequency $eV/\hbar$.

At this point, we make contact with previous calculations of the tunnelling current in [4]. If we ignore the weak θ-dependence of the vertex factor, the dynamics depend only on the azimuthal angle ϕ of the pseudospin. This is the same theory as in [4]. However, that work gives a *vanishing* tunnelling current at zero temperature. Our calculation is different from that one since we include quantum corrections involving multi-spinwave processes.

More quantitatively, the current is calculated in [4] from $I_\phi \sim \int d^2\boldsymbol{r}\, dt \exp(ieVt/\hbar - G_Q)$ where the quantum propagator G_Q is, in our notation:

$$G_Q(\boldsymbol{r}, t) = -i(v\hbar/2\pi S\rho_E)\left[(vt - ia)^2 - r^2\right]^{-1/2}. \quad (6)$$

We have explicitly included a lattice cutoff, $a \sim l_B$, that reflects the finite bandwidth of the spin waves. We recall that our parameters satisfy $(\hbar v/\rho_E S l_B) < 1$ to avoid quantum disordering. We can therefore perform the integral order by order in $(\hbar v/\rho_E S a)$, finding:

$$I_\phi = \frac{\exp(-eVa/\hbar v)}{V} \frac{D\tilde{\Delta}_{\text{SAS}}^2}{16\pi\hbar l_B^2 \rho_E} \frac{d^2}{d\Omega^2} I_0(2\sqrt{\Omega}) \quad (7)$$

where $\Omega = eV/4\pi S\rho_E$ and I_0 is a modified Bessel function. The leading terms in the voltage V are also the leading ones in $1/S$, justifying the expansion about the classical limit.

In an expansion in $eV/\rho_E S$, the leading term in eq. (7) coincides with eq. (5) if we ignore $X(\theta)$. In fact, the nth term in the expansion arises from the decay of a single quantum of the $q = 0$ mode into $n + 2$ finite-momentum spinwaves. Thus, the zero-temperature tunnelling current arises from multi-spinwave processes. These contributions have been ignored in [4] although they are, in principle, contained in the framework of that paper.

We now generalise to the case in which a magnetic field is applied in the plane of the bilayer. In that case, the tunnelling term in the Hamiltonian acquires a spatial dependence: $\Delta_{\text{SAS}} m^x \to \Delta_{\text{SAS}}[m^x \cos(Qx) + m^y \sin(Qx)]$,

where $Q = (eB_y d/\hbar c)^{1/2}$ is the wavelength associated with the in-plane field B_y. The spacing, d, between the two electron layers is approximately 30nm.

The tunnelling vertex now involves a change of momentum Q in the x-direction. A new process appears: a single $q = 0$ quantum may decay into a single spinwave (Fig. 2b). The contribution from this process leads to a δ function peak in the dissipation. The two-spinwave process contributes to the dissipation rate only for $eV > vQ$. The leading contributions to the current density are

$$
\begin{aligned}
I_{\theta > \theta_0, Q} = & \frac{D\tilde{\Delta}_{\text{SAS}}^2}{16\pi\hbar l_B^2} \times \Big[2\pi V^{-1}\delta(eV - vQ) + \\
& \frac{1}{2\rho_E S} \frac{e}{\sqrt{(eV)^2 - (vQ)^2}}\Theta(eV - vQ)\Big]
\end{aligned}
\tag{8}
$$

where $\Theta(x)$ is the step function. The peak at $eV = vQ$ is asymmetric. The lineshape is controlled by the multi-spinwave processes. For $eV < vQ$, there is no available spinwave channel for dissipation and so there is no tunnelling current: at larger voltages, the current decays as a power law. This is shown in Fig. 3, in which we have broadened the delta function by introducing low-momentum scattering with momentum spread σ_Q. This may arise from disorder or thermal effects. In the absence of these effects, the delta function will be broadened by the intrinsic lifetimes of the final states (for example, by further tunnelling or spinwave/spinwave scattering).

The treatment thus far has been valid only for voltages greater than $V_0 = 2\Delta_{\text{sw}}/e$. We now discuss charging voltages smaller than this value. In this regime, our thought experiment does not result in a steady tunnelling current. Instead, the current and charge imbalance oscillate coherently around zero with frequency $\Delta_{\text{sw}}/\hbar$. We now evaluate the rate at which these oscillations decay.

We treat the parameter θ perturbatively in this regime. In the Holstein-Primakov representation, the quadratic part of the Hamiltonian takes the form:

$$
H_{\theta < \theta_0}^{(0)} = \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} \beta_{\boldsymbol{q}}^\dagger \beta_{\boldsymbol{q}}
\tag{9}
$$

where $\omega_q = [(\Delta_{\text{SAS}} + \rho_E\gamma(\boldsymbol{q}))(D + \Delta_{\text{SAS}} + \rho_E\gamma(\boldsymbol{q}))]^{1/2}$ and $\beta_{\boldsymbol{q}}$ annihilates a spinwave with wavevector $\boldsymbol{q}$.

The initial state of our thought experiment has a charge imbalance. This corresponds to a condensate of bosons with $q=0$. These bosons have the small concentration $n_0/S = \theta^2[1 + (D/\Delta_{\text{SAS}})]^{1/2}/4\pi l_B^2$. The simplest dissipative process involves four $q=0$ quanta annihilating to form a pair of spinwaves with finite momenta. We defer details to a later paper[12]: the dissipation rate is

$$
\frac{\Gamma_{\theta < \theta_0}}{L^2} = \left(\frac{\theta}{\theta_0}\right)^8 \frac{\Delta_{\text{SAS}}^3 \Delta_{\text{sw}}^2}{512\pi\rho_E D^2 l_B^2 \hbar} f\left(\frac{\Delta_{\text{SAS}}}{D}\right)^2
\tag{10}
$$

where $f(x) = (1+x)^{-4}[3 - 4x - 8x^2 - x\sqrt{1 + 16x + 16x^2}]$ is

approximately equal to 3 since $\Delta_{\text{SAS}} \ll D$. This dissipation corresponds a decay of the density of $q = 0$ modes: $\dot{n} = -\Gamma(\theta)/4\hbar\Delta_{\text{sw}} L^2$. Since the density n is proportional to θ^2, we see that $d\theta^2/dt \sim -\theta^8$. The amplitude A of the oscillations in the tunnelling current is proportional to $\sin\theta$, and so it decays in time as a power law: $A(t) \sim 1/t^{1/6}$.

The dissipation is very weak at small initial angles partly due to the kinematic constraint for a 4-spinwave collision. One might expect stronger dissipation in the presence of disorder or inelastic scattering, e.g., with phonons or charged quasiparticles. Nevertheless, the multi-spinwave processes provide an intrinsic damping mechanism that has not previously been identified.

We note that this treatment does not lead to a state with a direct tunnelling current at $V < V_0$. Therefore we cannot address the lineshape of the zero-bias peak in dI/dV. It is worth noting that this feature has sharpened with improved experimental conditions. It may be controlled by low-energy states introduced by disorder or by effects arising from coupling to external leads. An approach similar to [5] may be needed.

In conclusion, we find that the zero-temperature tunnelling current has a strong $1/V$ peak arising from the decoherence of Josephson-like oscillations by the generation of electron density fluctuations in the bilayer. We also predict a non-trivial lineshape for the dispersing feature that appears in the presence of an in-plane field. These effects resemble features reported in experiments[2].

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